Exercícios

1. $\lim\limits_{x \to 1} \dfrac{x^2+2x-3}{x^2-1} = \text{"}\dfrac{0}{0}\text{"}$ $\quad$ $\text{Dom } f = \mathbb{R} \setminus \{-1; 1\}$

$\lim\limits_{x \to 1} \dfrac{(x+3)(x-1)}{(x-1)(x+1)}$

$= \lim\limits_{x \to 1} \dfrac{x+3}{x+1} = \dfrac{4}{2} = 2$

2. $\lim\limits_{x \to \frac{1}{2}} \dfrac{2x^2+3x-2}{4x^2-1} = \text{"}\dfrac{0}{0}\text{"}$ $\quad$ $\text{Dom } f = \mathbb{R} \setminus \{-\frac{1}{2}; \frac{1}{2}\}$

$N: \Delta = 9 + 16 = 25$

$x_{1,2} = \dfrac{-3 \pm 5}{4} = \begin{cases} -2 \\ \frac{1}{2} \end{cases}$

$\lim\limits_{x \to \frac{1}{2}} \dfrac{2 \cdot (x+2) \cdot (x-\frac{1}{2})}{(2x-1)(2x+1)}$

$= \lim\limits_{x \to \frac{1}{2}} \dfrac{x+2}{2x+1} = \dfrac{5}{2} \cdot \dfrac{1}{2} = \dfrac{5}{4}$

3. $\lim\limits_{x \to 2} \dfrac{x^2+x-6}{x^2-4x+4} = \text{"}\dfrac{0}{0}\text{"}$ $\quad$ $\text{Dom } f = \mathbb{R} \setminus \{2\}$

$= \lim\limits_{x \to 2} \dfrac{(x-2)(x+3)}{(x-2)^2}$

$= \lim\limits_{x \to 2} \dfrac{x+3}{x-2} = \begin{cases} \lim\limits_{x \to 2^-} \dfrac{x+3}{x-2} = \dfrac{5}{0^-} = -\infty \\ \lim\limits_{x \to 2^+} \dfrac{x+3}{x-2} = \dfrac{5}{0^+} = +\infty \end{cases}$

		2	
$x-2$	$-$	0	$+$

4. $\lim\limits_{x \to -1} \dfrac{3x^2-2x-5}{-x^2-3x-2} = \text{"}\dfrac{0}{0}\text{"}$ $\quad$ $\text{Dom } f = \mathbb{R} \setminus \{-2; -1\}$

$N: \Delta = 4 + 60 = 64$

$x_{1,2} = \dfrac{2 \pm 8}{6} = \begin{cases} \frac{5}{3} \\ -1 \end{cases}$

$D: \Delta = 9 - 8 = 1$

$x_{1,2} = \dfrac{3 \pm 1}{-2} = \begin{cases} -2 \\ -1 \end{cases}$

$\lim\limits_{x \to -1} \dfrac{3 \cdot (x-\frac{5}{3}) \cdot (x+1)}{-(x+2)(x+1)} = \lim\limits_{x \to -1} \dfrac{3x-5}{-(x+2)} = +8$

5. $\lim_{x \to -\frac{1}{2}} \frac{2x^2 - 13x - 7}{4x^2 + 4x + 1} =$ Dom $f = \mathbb{R} \setminus \{-\frac{1}{2}\}$

N: $\Delta = 169 + 56 = 225$

$x_{1,2} = \frac{13 \pm 15}{4} = \begin{cases} 7 \\ -\frac{1}{2} \end{cases}$

$\lim_{x \to -\frac{1}{2}} \frac{2 \cdot (x-7)(x+\frac{1}{2})}{(2x+1)^2}$

$= \lim_{x \to -\frac{1}{2}} \frac{x-7}{2x+1} = \frac{-\frac{15}{2}}{0}$ $\begin{cases} \lim_{x \to -\frac{1}{2}^-} \frac{-\frac{15}{2}}{0^-} = +\infty \\ \lim_{x \to -\frac{1}{2}^+} \frac{-\frac{15}{2}}{0^+} = -\infty. \end{cases}$

$$\begin{array}{c|ccc} & & -\frac{1}{2} & \\ \hline 2x+1 & - & 0 & + \end{array}$$

6. $\lim_{x \to 1} \frac{3x^3 - 4x^2 - x + 2}{3x^2 - 6x + 3} = \text{"}\frac{0}{0}\text{"}$ Dom $f = \mathbb{R} \setminus \{1\}$

$$\begin{array}{c|ccc|c} & 3 & -4 & -1 & 2 \\ 1 & & 3 & -1 & -2 \\ \hline & 3 & -1 & -2 & 0 \end{array}$$

$$\begin{array}{c|cc|c} 1 & & 3 & 2 \\ \hline & 3 & 2 & 0 \end{array}$$

$\lim_{x \to 1} \frac{(x-1)^2 \cdot (3x+2)}{3 \cdot (x-1)^2}$

$= \lim_{x \to 1} \frac{3x+2}{3} = \frac{5}{3}.$

7. $\lim_{x \to -1} \frac{x^3 + 3x^2 + 3x + 1}{x^3 - 3x - 2} = \text{"}\frac{0}{0}\text{"}$ Dom $f = \mathbb{R} \setminus \{-1; 2\}$

D:
$$\begin{array}{c|ccc|c} & 1 & 0 & -3 & -2 \\ -1 & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & 0. \end{array}$$

$$\begin{array}{c|cc|c} -1 & & -1 & 2 \\ \hline & 1 & -2 & 0 \end{array}$$

$\lim_{x \to -1} \frac{(x+1)^3}{(x+1)^2(x-2)} = \lim_{x \to -1} \frac{x+1}{x-2} = 0.$